



Mathematical Fluency and Number Sense: Techniques, Access and Sustainability for All Students

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Essential Questions to Answer Today

- What are number sense and fluency, how are they related and why are they important?
- What activities build fluency and number sense together?
- What are important look-fors in the classroom?
- What role does developing fluency and number sense play with intervention and special education?

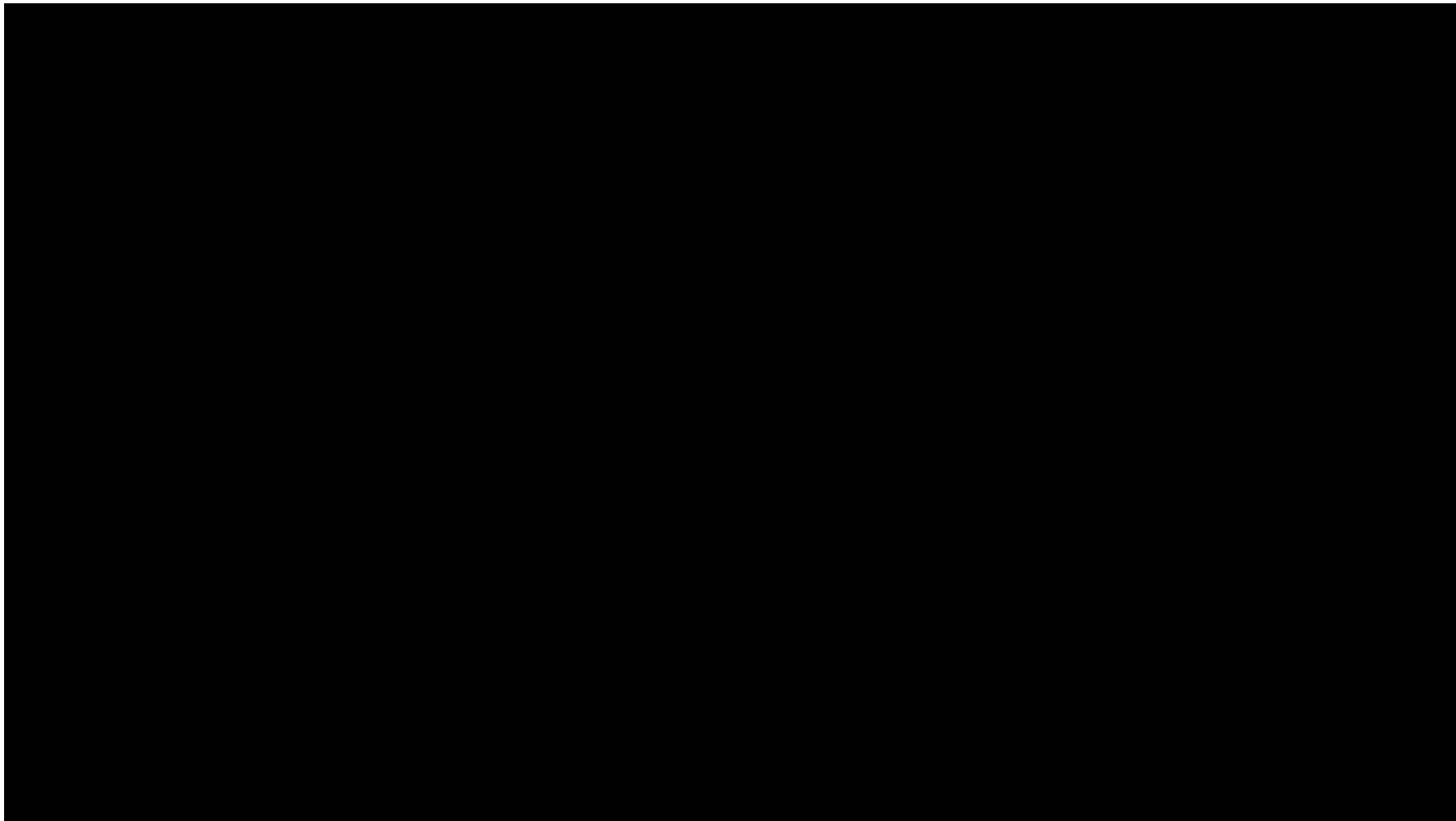


Fluency – There is always your fingers

Kid Snippets – "Math Class"

<https://www.youtube.com/watch?v=KdxEAt91D7k>





Kid Snippets – "Math Class"

What Is Fluency?

. . . procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) . . .

CCSSM, p. 6



Number Sense

- *Fluidity and flexibility with numbers*
- *Sense of what numbers mean*
- *Ability to perform mental mathematics and to look at the world and make comparisons*

Gersten & Chard 2001

Phone Number Equation

Using your phone number, without the area code, create at least one equation

- Use any combination of operations
- Create an expression with the first three numbers equal to an expression created with the last four numbers.

Example: 328-6537 $\rightarrow 8 \times (3 + 2) = (6 \times 5) + (3 + 7)$

Example: 328-6537 $\rightarrow (3 - 2) \times 8 = (7 - 6) \times (5 + 3)$

NAEP 4th Grade: Closest to $\frac{1}{2}$

Which fraction has a value closest to $\frac{1}{2}$?

A. $\frac{5}{8}$

B. $\frac{1}{6}$

C. $\frac{2}{2}$

D. $\frac{1}{5}$

Which is the correct answer?

Which do you think is the most popular incorrect answer and why?

25% answered correctly (A)

40% chose C

Why Worry About Fluency?

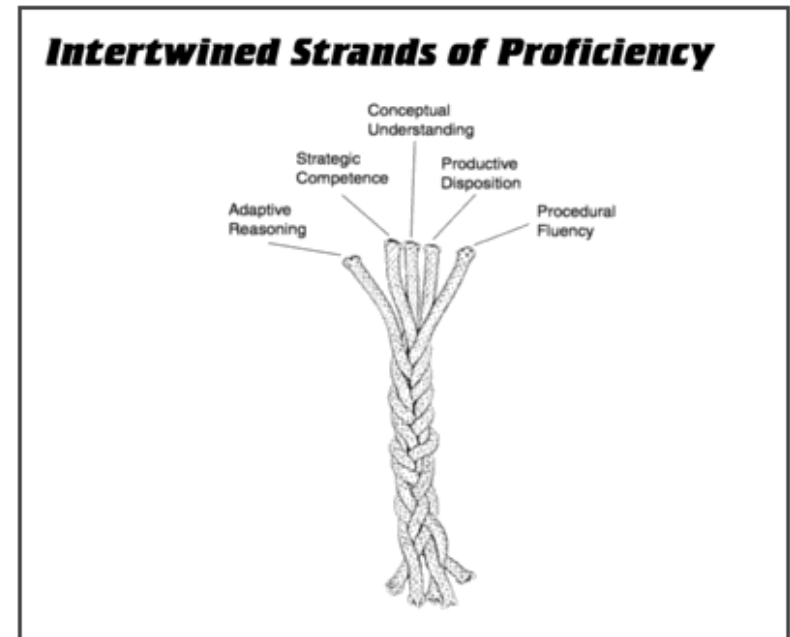
- Real world
- Standards
- Assessments
- Impact on learning



Five Proficiency Strands in Mathematics

- Conceptual understanding
- Strategic competence
- Adaptive reasoning
- Productive disposition
- Procedural fluency

Box 4-1



National Research Council 2001

Fluency Chart for K–8

Grade	Required Fluency
K	Add and subtract within 5
1	Add and subtract within 10
2	Add and subtract within 20 (mentally) Add and subtract within 100
3	Multiply and divide within 100 Add and subtract within 1,000
4	Add and subtract multidigit whole numbers using standard algorithms
5	Multiply multidigit whole numbers using standard algorithm
6	Add, subtract, multiply, and divide multidigit numbers (including decimals) using standard algorithms
6-8	Compute with positive and negative fractions and decimals

Assessments and Fluency

Assessments typically include three key areas:

- Facts and procedures
- Concepts
- Applications

How Many Number Facts?

(Handout p. 5)

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

Limits of Working Memory

$$7(x + 8) = 7x + 56$$

Capacity and Time

Age (Years)	Average Capacity Range (Chunks of Information)	Average Time Limit for Retention
Younger than 5	2 ± 1	Unknown
From 5 to 14	5 ± 2	5 to 10 minutes
Older than 14	7 ± 2	10 to 20 minutes

David A. Sousa, *How the Brain Learns Mathematics*, 2008

Making Room in Working Memory



Our ability to think would be limited indeed if there were not ways to overcome the space constraint of working memory. One of the more important mechanisms is the development of automaticity. When cognitive processes . . . become automatic, they demand very little space in working memory, they occur rapidly, and they often occur without conscious effort.

Daniel Willingham, *American Educator*, Spring 2004

Recommendations from Research

- *Provide 10 minutes of daily practice to strengthen needed fluency with facts and procedures.*

Gersten et al., (2009)

- *Distributed or spaced practice, repeated practice of previously learned knowledge over “a long period of time,” has a high effect size of 0.71.*

John Hattie, Douglas Fisher, and Nancy Frey (2017)

- *Fluency with whole numbers and fractions are part of a critical foundation for learning algebra.*

National Mathematics Advisory Panel (2008)

Rote and Elaborate Rehearsal

- **Rote rehearsal:** Memorization without continuing to think through an idea or fact.
- **Elaborate rehearsal:** Making sense of ideas and information. The learner processes and reprocesses information to connect it together, to connect it to prior learning, and to assign meaning to it. Elaborate rehearsal is necessary for students to probe the deeper meaning and interrelationships of mathematical concepts.

Why We Need Elaborate Rehearsal



Memories are formed as the residue of thought. You remember what you think about, but not every fleeting thought—only those matters to which you really devote some attention.

Daniel Willingham 2008

Fluency Activities with Number Sense in Mind



Making 24

Use the four numbers and any combination of math operations to get a result of 24.

Use **1, 1, 4** and **6** to make 24: $1 \times 1 \times 4 \times 6 = 24$

Use **8, 8, 9**, and **11** to make 24: $9 / (11 - 8) \times 8 = 24$

You try it: use **6, 6, 11**, and **13** to make 24:

$$13 \times (6 / 6) + 11 = 24$$

Great website: <https://www.4nums.com/>

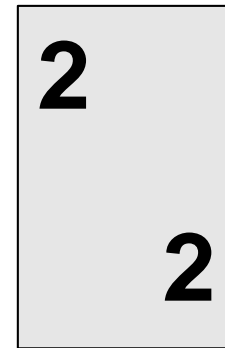
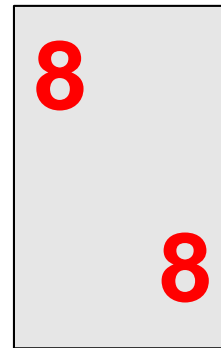
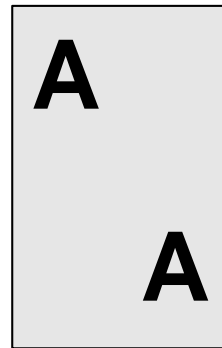
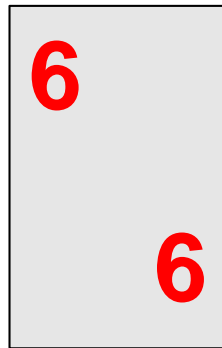
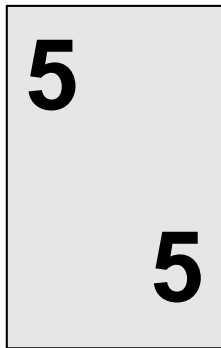
Card Games

Examples

- Integer Addition War
- Hit the Target
- Addition War
- Fraction War

Hit the Target

Target is 12



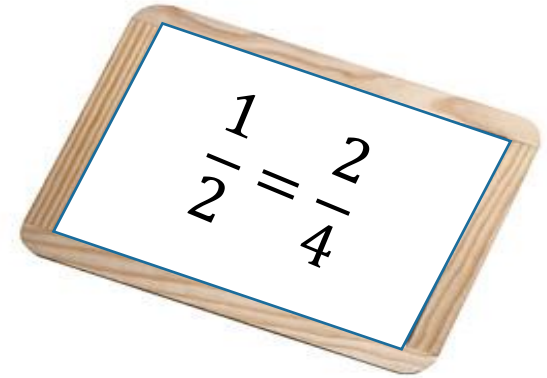
$$5 + 6 + 1 \qquad 2 \times 6 \qquad 2(5 + 1)$$
$$(8 - 6) \times 5 \times 1 + 2$$

KenKenPuzzle.com

3+	1	2	3
7+	3	1	3+
2	2	3	1

7+	4+		3+
	6+	4	
1		5+	7+
3+			

Individual White Board Review



1. $\frac{1}{2} = \frac{\quad}{4}$

5. $\frac{3}{4} = \frac{\quad}{20}$

2. $\frac{1}{2} = \frac{\quad}{10}$

6. $\frac{1}{4} = \frac{3}{\quad}$

3. $\frac{1}{4} = \frac{\quad}{8}$

7. $\frac{3}{9} = \frac{\quad}{3}$

4. $\frac{1}{4} = \frac{\quad}{20}$

8. $\frac{4}{12} = \frac{2}{\quad}$

SPRINTS

A

Number Correct: _____

Write the Missing Factor

1.	$10 = 5 \times \underline{\quad}$	
2.	$10 = 2 \times \underline{\quad}$	
3.	$8 = 4 \times \underline{\quad}$	
4.	$9 = 3 \times \underline{\quad}$	
5.	$6 = 2 \times \underline{\quad}$	
6.	$6 = 3 \times \underline{\quad}$	
7.	$12 = 6 \times \underline{\quad}$	
8.	$12 = 3 \times \underline{\quad}$	
9.	$12 = 4 \times \underline{\quad}$	
10.	$12 = 2 \times 2 \times \underline{\quad}$	
11.	$12 = 3 \times 2 \times \underline{\quad}$	
12.	$20 = 5 \times 2 \times \underline{\quad}$	
13.	$20 = 5 \times 2 \times \underline{\quad}$	
14.	$16 = 8 \times \underline{\quad}$	
15.	$16 = 4 \times 2 \times \underline{\quad}$	
16.	$24 = 8 \times \underline{\quad}$	
17.	$24 = 4 \times 2 \times \underline{\quad}$	
18.	$24 = 4 \times \underline{\quad} \times 2$	
19.	$24 = 3 \times 2 \times \underline{\quad}$	
20.	$24 = 3 \times \underline{\quad} \times 2$	
21.	$6 \times 4 = 8 \times \underline{\quad}$	
22.	$6 \times 4 = 4 \times 2 \times \underline{\quad}$	

23.	$28 = 7 \times \underline{\quad}$	
24.	$28 = 2 \times 2 \times \underline{\quad}$	
25.	$28 = 2 \times \underline{\quad} \times 2$	
26.	$28 = \underline{\quad} \times 2 \times 2$	
27.	$36 = 3 \times 3 \times \underline{\quad}$	
28.	$9 \times 4 = 3 \times 3 \times \underline{\quad}$	
29.	$9 \times 4 = 6 \times \underline{\quad}$	
30.	$9 \times 4 = 3 \times 2 \times \underline{\quad}$	
31.	$8 \times 6 = 4 \times \underline{\quad} \times 2$	
32.	$9 \times 9 = 3 \times \underline{\quad} \times 3$	
33.	$8 \times 8 = \underline{\quad} \times 8$	
34.	$7 \times 7 = \underline{\quad} \times 7$	
35.	$8 \times 3 = \underline{\quad} \times 6$	
36.	$16 \times 2 = \underline{\quad} \times 4$	
37.	$2 \times 18 = \underline{\quad} \times 9$	
38.	$28 \times 2 = \underline{\quad} \times 8$	
39.	$24 \times 3 = \underline{\quad} \times 9$	
40.	$6 \times 8 = \underline{\quad} \times 12$	
41.	$27 \times 3 = \underline{\quad} \times 9$	
42.	$12 \times 6 = \underline{\quad} \times 8$	
43.	$54 \times 2 = \underline{\quad} \times 12$	
44.	$9 \times 13 = \underline{\quad} \times 39$	

It's not a race.

It's not a test.

It's practice.

*It's learning
strategies and
building fluency.*

**EUREKA
MATH™**


Lesson 1:
Date:

Make equivalent fractions with the number line, the area model,
and numbers.
3/15/15

engage^{ny}

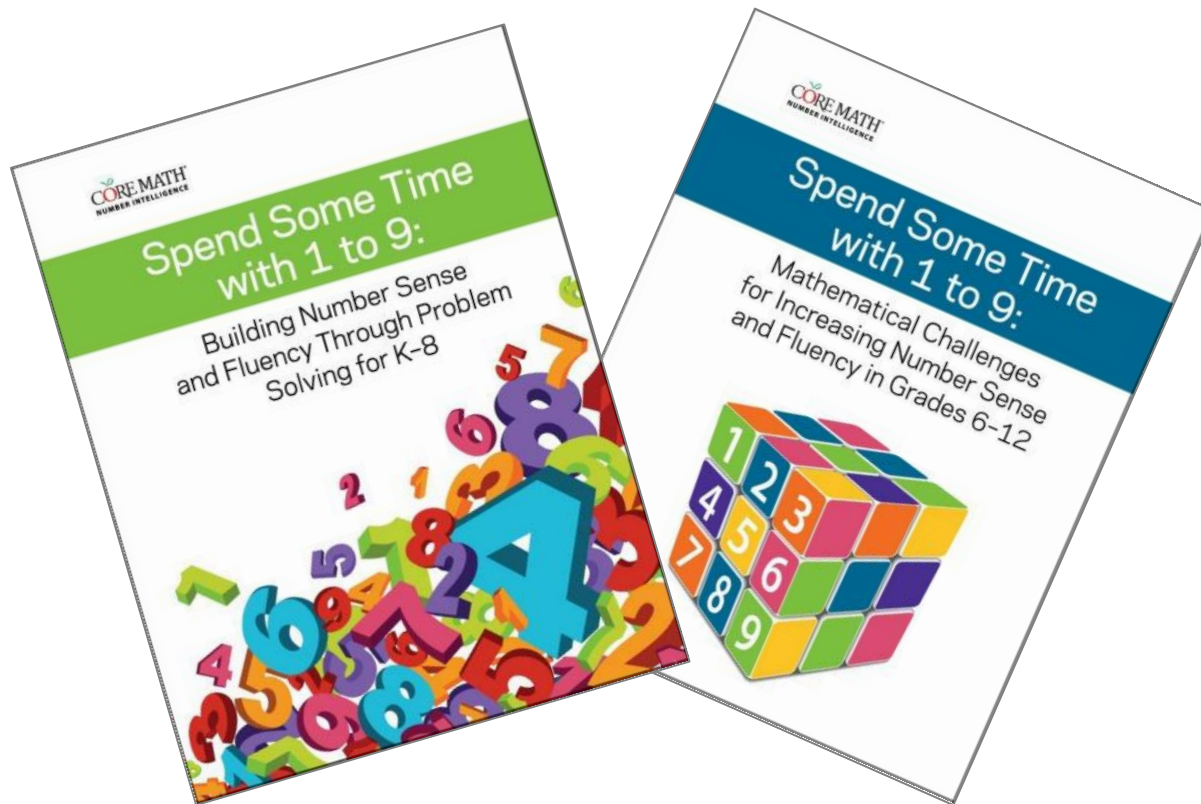
19

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Spend Some Time with 1 to 9

Problem-solving and practice to build fluency and number sense. www.Corelearn.com



Create Equations with the Digits 1–9

Create as many equations as you can with the digits 1 – 9:

- Use some or all of the **digits** in each **equation**.

$$8 \div 4 = 5 - 3$$

$$6 \times 7 = 42$$

$$7 \times 5 + 8 - 6 = 29 + 1 + 3 + 4$$

- Do not use any digit more than once within any equation.

non-example: ~~$8 \div 4 + 3 = 7 - 3 + 1$~~

- Do not use the digit zero.

non-example: ~~$16 \div 2 = 40 \div (8 - 3)$~~

- You may use any math operations.

Add, subtract, multiply, divide, exponents, etc.

Mystery Math Grids

X

12	20	32
6	10	16
18	15	48

+ **3** **5** **8**

9	12	14	17
2	5	7	10
1	4	6	9

You try it. What are the outside numbers for this Mystery Math Grid?

X	[]		
[]	6	21	24
	10	35	40
	12	42	48

Connections to Intervention and Special Education

- Research on priority
- Balance between conceptual and procedural (remember the five strands of proficiency)
- Application also builds fluency and number sense - Include reasoning and problem solving

Common Traits of Fluency + Number Sense Activities

- Build fluency
- Build number sense
- Doable – Quick, accessible, “successible”
- Adaptable
- Durable

Common Issues with Fluency + Number Sense Activities

- Time constraints in lessons – fitting in fluency
- Finding fluency + number sense activities
- First few steps are the hardest
 - Teachers finding the right way to get students started
 - Teachers and students learning new activities and routines

Look-fors in Classrooms

- Short regular doses of fluency activities.
- Activities vary by intent – Fluency & Fluency+
- Activities vary by type – different types of fluency and different types of fluency+ activities
- Activities do not unintentionally take over lessons.
- Students are highly engaged. Not drill and kill , but rather **strive and thrive**.
- Questioning from teachers to get students to think
- Number lines

Favorite Fluency + Number Sense Activities

- Oral counting
- Card games
- Number Talks
- Mystery Math Grids
- Individual White Boards
- Spend Some Time with 1 to 9
- KenKen Puzzles (kenkenpuzzles.com)
- Sprints (Bill Davidson, EngageNY/Eureka Math)
- Make 24 (<https://www.coolmathgames.com/0-make-24>) & <https://www.4nums.com/game/difficulties/>

Let's Connect!

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If you'd like CORE to come to your school to provide instructional coaching and other professional learning services, please get in touch.



Webinar Bonus!

Spend Some Time with 1 to 9
FREE samplers

www.corelearn.com/resources/free-materials