



Consortium on Reaching Excellence in Education®

Your Implementation Partner for Literacy and Math Achievement

Fluency and Number Sense

Handout
NABE 2019

Presented by

Dean Ballard

Director of Mathematics

Consortium on Reaching Excellence in Education, CORE Inc.

Recommendations from Research

- *Provide 10 minutes of daily practice to strengthen needed fluency with facts and procedures (Gersten et al. 2009)*
- *Distributed or spaced practice, repeated practice of previously learned knowledge over “a long period of time,” has a high effect size of 0.71 (Hattie, Fisher, & Frey 2017)*
- *Fluency with whole numbers and fractions are part of a critical foundation for learning algebra (National Mathematics Advisory Panel 2008).*

Rote rehearsal: Memorization without continuing to think through an idea or fact.

Elaborate rehearsal: Making sense of ideas and information. The learner processes and reprocesses information to connect it together, to connect it to prior learning, and to assign meaning to it. Elaborate rehearsal is necessary for students to probe the deeper meaning and interrelationships of mathematical concepts.

Sousa in *How the Brain Learns Math* 2007

Our ability to think would be limited indeed if there were not ways to overcome the space constraint of working memory. One of the more important mechanisms is the development of automaticity. When cognitive processes . . . become automatic, they demand very little space in working memory, they occur rapidly, and they often occur without conscious effort.

Daniel Willingham 2004

Memories are formed as the residue of thought. You remember what you think about, but not every fleeting thought—only those matters to which you really devote some attention.

Daniel Willingham 2008

Phone Number Problem

- Using your phone number, without the area code, create at least one equation.
 - Use any combination of operations.
 - Create an expression with the first three numbers equal to an expression created with the last four numbers.

Example: 832-3657 $\rightarrow 8 \times (3 + 2) = (6 \times 5) + (3 + 7)$

Example: 832-3657 $\rightarrow (3 - 2) \times 8 = (7 - 6) \times (5 + 3)$

7

www.corelearn.com

© 2019 Consortium on Reaching Excellence in Education, Inc.

CORE MATH

Phone Number Problem

Using your phone number, without the area code, create at least one equation.

- Use any combination of operations.
- Create an expression with the first three numbers equal to an expression created with the last four numbers.

Fluency Chart
Fluencies in the Common Core State Standards for Mathematics

Grade	Required Fluency
K	Add and subtract within 5
1	Add and subtract within 10
2	Add and subtract within 20 (mentally) Add and subtract within 100
3	Multiply and divide within 100 Add and subtract within 1,000
4	Add and subtract multidigit whole numbers using standard algorithms
5	Multiply multidigit whole numbers using standard algorithm
6	Add, subtract, multiply, and divide multidigit numbers (incl. decimals) using standard algorithms
6-8	Compute with positive and negative fractions and decimals

How Many Products Are There to Memorize? (CCSSM: 3.OA.7)

Learning multiplication number facts or products to the point of instant recall may at first appear to require learning 121 or more facts for all combinations of factors from 0 to 10. However, use of number properties, counting strategies, and number relationships, such as the commutative property, etc., can significantly reduce this number as well as make the process of memorizing the products easier and more meaningful.

1. Fill in all 121 products or multiplication facts in the table below. Each cell is the intersection of two factors – the factor at the top of the column and the factor at the beginning of the row.
2. Make a list of the strategies and properties that will reduce the number of products to be memorized. Explain how each strategy or property can be applied, and how many products on the chart below it can be applied to.
3. How many products need to be memorized? Justify your answer.

x	0	1	2	3	4	5	6	7	8	9	10
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											

Multiply

1	$1 \times 2 =$		23	$3 \times 2 =$	
2	$1 \times 0.2 =$		24	$0.3 \times 2 =$	
3	$1 \times 0.02 =$		25	$0.3 \times 0.2 =$	
4	$2 \times 1 =$		26	$0.3 \times 0.02 =$	
5	$2 \times 0.1 =$		27	$6 \times 8 =$	
6	$2 \times 0.01 =$		28	$6 \times 0.8 =$	
7	$2 \times 3 =$		29	$0.6 \times 0.8 =$	
8	$2 \times 0.3 =$		30	$0.6 \times 0.08 =$	
9	$2 \times 0.03 =$		31	$5 \times 8 =$	
10	$2 \times 4 =$		32	$5 \times 0.8 =$	
11	$2 \times 0.4 =$		33	$0.5 \times 0.8 =$	
12	$2 \times 0.04 =$		34	$0.5 \times 0.08 =$	
13	$4 \times 3 =$		35	$6 \times 0.9 =$	
14	$4 \times 0.3 =$		36	$0.6 \times 9 =$	
15	$4 \times 0.03 =$		37	$0.6 \times 0.9 =$	
16	$2 \times 5 =$		38	$0.6 \times 0.09 =$	
17	$2 \times 0.5 =$		39	$0.5 \times 0.4 =$	
18	$2 \times 0.05 =$		40	$0.5 \times 0.04 =$	
19	$3 \times 0.7 =$		41	$0.5 \times 0.004 =$	
20	$5 \times 0.7 =$		42	$0.7 \times 0.9 =$	
21	$4 \times 0.07 =$		43	$0.07 \times 0.09 =$	
22	$3 \times 0.07 =$		44	$0.07 \times 0.009 =$	

Multiply

1	$3 \times 2 =$		23	$3 \times 3 =$	
2	$3 \times 0.2 =$		24	$3 \times 0.3 =$	
3	$3 \times 0.02 =$		25	$0.3 \times 0.3 =$	
4	$2 \times 3 =$		26	$0.3 \times 0.03 =$	
5	$2 \times 0.3 =$		27	$7 \times 8 =$	
6	$2 \times 0.03 =$		28	$7 \times 0.8 =$	
7	$2 \times 2 =$		29	$0.7 \times 0.8 =$	
8	$2 \times 0.2 =$		30	$0.7 \times 0.08 =$	
9	$2 \times 0.02 =$		31	$5 \times 9 =$	
10	$3 \times 5 =$		32	$5 \times 0.9 =$	
11	$3 \times 0.05 =$		33	$0.5 \times 0.9 =$	
12	$5 \times 0.03 =$		34	$0.5 \times 0.09 =$	
13	$6 \times 4 =$		35	$8 \times 0.9 =$	
14	$6 \times 0.4 =$		36	$0.8 \times 9 =$	
15	$6 \times 0.04 =$		37	$0.8 \times 0.9 =$	
16	$6 \times 5 =$		38	$0.8 \times 0.09 =$	
17	$6 \times 0.5 =$		39	$0.8 \times 0.8 =$	
18	$6 \times 0.05 =$		40	$0.8 \times 0.08 =$	
19	$3 \times 0.7 =$		41	$0.8 \times 0.008 =$	
20	$4 \times 0.7 =$		42	$0.9 \times 0.9 =$	
21	$4 \times 0.07 =$		43	$0.09 \times 0.09 =$	
22	$5 \times 0.07 =$		44	$0.09 \times 0.009 =$	

Solve this Mystery Grid

x

6	21	24
10	35	40
12	42	48

Create you own and challenge a partner

Create you own and challenge a partner

Spend Sum Time with 1 to 9

1
2
3
4
5
6
7
8
9

Use the digits 1 to 9 to fill in the boxes in each set of three equations below to make the equations true. You cannot use a number more than once within the boxes in each set of three equations. You will notice that each row as three identical sets of equations. Each set in each problem has the same three equations.

Your goal is to fill in each set in the row so that the answer for each set is different than the answers for the other two sets in the same row in at least one way. Some fill-in numbers may be the same, but at least one fill-in must be different.

For example, together let's find different solutions for a set of three equations shown below.

$$\square + 2 = 5$$

$$\square + \square = 7$$

$$\square + 4 = 6 + \square$$

$$\square + 2 = 5$$

$$\square + \square = 7$$

$$\square + 4 = 6 + \square$$

$$\boxed{3} + 2 = \square$$

$$\boxed{2} + \boxed{5} = \square + \square$$

$$\boxed{9} + 4 = \square + \boxed{7} \quad \square$$

1.

$$\square + 2 = 7$$

$$\square + \square = 8$$

$$\square + 3 = 6 + \square$$

$$\square + 2 = 7$$

$$\square + \square = 8$$

$$\square + 3 = 6 + \square$$

$$\square + 2 = 7$$

$$\square + \square = 8$$

$$\square + 3 = 6 + \square$$

2.

$$\square + 3 = 7$$

$$\square + \square = 5$$

$$\square + 3 = 1 + \square$$

$$\square + 3 = 7$$

$$\square + \square = 5$$

$$\square + 3 = 1 + \square$$

$$\square + 3 = 7$$

$$\square + \square = 5$$

$$\square + 3 = 1 + \square$$

3.

$$\square + 8 = 9$$

$$\square + \square = 7$$

$$\square + 2 = 5 + \square$$

$$\square + 8 = 9$$

$$\square + \square = 7$$

$$\square + 2 = 5 + \square$$

$$\square + 8 = 9$$

$$\square + \square = 7$$

$$\square + 2 = 5 + \square$$

Broken Calculator

Scenario

Students are given a challenge to make each number on a given list of numbers using just the keys that work on a "broken" calculator. The keys that work are shown on the calculator.



www.mathsisfun.com/games/broken-calculator.html

49

www.corelearn.com

© 2019 Consortium on Reaching Excellence in Education, Inc.

CORE.MATH

Broken Calculator

A calculator only has the 4, 7, 9, \times , $-$, min, MR, and = buttons working. Use it to create each of the following numbers:

1	11
2	12
3	19
5	25
8	27

min			MR	
				AC
4			\times	
7		9	$-$	
				=

Go to www.mathsisfun.com/games/broken-calculator.html to play Broken Calculator online and find other great resources.