CORE Middle School Math Academy Alignment with Common Core State Standards

CORE Middle School Math Academy emphasizes throughout each day the importance of teaching along the five proficiency strands identified by the National Research Council in *Adding It Up* (2001):

- Conceptual understanding
- Strategic competence
- Adaptive reasoning
- Productive disposition
- Procedural fluency

These correspond to the eight mathematical practices emphasized in the Common Core State Standards for all grade levels, as follows:

1. Make sense of problems and persevere in solving them. (*Procedural fluency*)
2. Reason abstractly and quantitatively. (*Adaptive reasoning*)
3. Construct viable arguments and critique the reasoning of others. (*Adaptive reasoning*)
4. Model with mathematics. (*Conceptual understanding and strategic competence*)
5. Use appropriate tools strategically. (*Strategic competence*)
6. Attend to precision. (*Procedural fluency*)
7. Look for and make use of structure. (*Conceptual understanding and strategic competence*)
8. Look for and express regularity in repeated reasoning. (*Conceptual understanding, strategic competence, and procedural fluency*)
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<th>Focus Area</th>
<th>Standards</th>
<th>Corresponding CORE Service</th>
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<tr>
<td>Number and Operations in Base Ten</td>
<td>Use place value understanding to perform multidigit arithmetic</td>
<td>This session serves as an intervention for struggling adolescent learners focusing on operations with multidigit numbers, mental math, and number sense.</td>
<td>CORE Math Academy for Middle Schools, Sessions 1 and 5</td>
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<td></td>
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<td>• Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count. Explain the reasoning used.</td>
<td>Number Sense and Operations</td>
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<td>• Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. Relate the strategy to a written method and explain the reasoning used.</td>
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<td>• Fluently add and subtract multidigit whole numbers using the standard algorithm.</td>
<td>Entering Algebra</td>
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<td></td>
<td>• Fluently multiply multidigit whole numbers using the standard algorithm.</td>
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<td>• Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.</td>
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<td>• Add, subtract, multiply, and divide decimals to hundredths.</td>
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### Common Core Strand

**Operations and Algebraic Thinking**

### Focus Area

**Represent and solve problems involving using the four operations with whole numbers**

### Standards

This session serves as an intervention for struggling adolescent learners focusing on operations with integers, operations with multidigit numbers, and writing and solving simple equations.

- Interpret products and quotients of whole numbers.
- Use multiplication and division within 100 to solve word problems.
- Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.
- Multiply or divide to solve word problems involving multiplicative comparison.
- Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
- Apply properties of operations to add and subtract.
- Understand subtraction as an unknown-addend problem.
- Apply properties of operations as strategies to multiply and divide.
- Understand division as an unknown-factor problem.
- Fluently multiply and divide within 100.
- Find all factor pairs for a whole number in the range 1–100.
- Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations.

### Corresponding CORE Service

**CORE Math Academy for Middle Schools, Sessions 1 and 3**

**Number Sense and Operations**

**Multiplicative Thinking**
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| Operations and Algebraic Thinking (cont.) | Represent and solve problems involving using the four operations with whole numbers | • Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.   
• Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.   
• Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.   
• Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.   
• Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. | CORE Math Academy for Middle Schools, Sessions 1 and 3   
Number Sense and Operations   
Multiplicative Thinking |
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| Number and Operations: Fractions | Develop understanding of fractions, fraction equivalence, and operations with fractions | This session serves as an intervention for struggling adolescent learners focusing on understanding and operations with fractions.  
- Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by a parts of size $1/b$.  
- Understand a fraction as a number on the number line. Represent fractions on a number line diagram.  
- Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.  
- Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models.  
- Compare two fractions with different numerators and different denominators.  
- Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.  
- Use decimal notation for fractions with denominators 10 or 100.  
- Compare two decimals to hundredths by reasoning about their size.  
- Add and subtract fractions with unlike denominators.  
- Solve word problems involving addition and subtraction of fractions referring to the same whole. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.  
- Develop understanding of fractions as numbers.  
- Extend understanding of fraction equivalence and ordering.  
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. | CORE Math Academy for Middle Schools, Session 2  
Fractions |
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<th>Number and Operations: Fractions (cont.)</th>
<th>Develop understanding of fractions, fraction equivalence, and operations with fractions</th>
<th>CORE Math Academy for Middle Schools, Session 2</th>
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<td>• Understand a fraction a/b with a &gt; 1 as a sum of fractions 1/b.</td>
<td>Fractions</td>
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<td>• Use equivalent fractions as a strategy to add and subtract fractions.</td>
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<td>• Interpret a fraction as division of the numerator by the denominator.</td>
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<td>• Apply and extend previous understandings of multiplication to multiply fractions.</td>
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<td>• Interpret multiplication as scaling.</td>
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<td>• Solve real-world problems involving multiplication of fractions and mixed numbers.</td>
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<td>• Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</td>
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<td>• Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.</td>
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| **The Number System** | Apply and extend previous understandings of multiplication and division to divide fractions by fractions | • Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions.  
• Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers. Represent addition and subtraction on a horizontal or vertical number line diagram.  
• Describe scenarios where opposite quantities combine to make 0.  
• Understand subtraction of rational numbers as adding the additive inverse, \( p - q = p + (-q) \).  
• Apply properties of operations as strategies to add and subtract rational numbers. | CORE Math Academy for Middle Schools, Sessions 1, 2 and 5 |
| | Compute fluently with multidigit numbers and find common factors and multiples | • Fluently divide multidigit numbers using the standard algorithm.  
• Fluently add, subtract, multiply, and divide multidigit decimals using the standard algorithm for each operation.  
• Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. | Number Sense and Operations  
Fractions  
Entering Algebra |
| | Apply and extend previous understandings of numbers to the system of rational numbers | • Understand that positive and negative numbers are used together to describe quantities having opposite directions or values, and use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.  
• Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. | |
### Core Math Academy for Middle Schools, Session 3

#### Multiplicative Thinking

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| Ratios and Proportional Relationships | Understand ratio concepts and use ratio reasoning to solve problems | - Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.  
- Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number-line diagrams, or equations).  
- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and use tables to compare ratios.  
- Solve unit rate problems including those involving unit pricing and constant speed.  
- Find a percent of a quantity as a rate per 100. Solve problems involving finding the whole, given a part and the percent.  
- Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.  
- Recognize and represent proportional relationships between quantities.  
- Decide whether two quantities are in a proportional relationship.  
- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.  
- Represent proportional relationships by equations.  
- Explain what a point \((x, y)\) on the graph of a proportional relationship means in terms of the situation, with special attention to the points \((0, 0)\) and \((1, r)\) where \(r\) is the unit rate.  
- Use proportional relationships to solve multistep ratio and percent problems. | CORE Math Academy for Middle Schools, Session 3 |
Math Services Align with the Common Core State Standards
Mathematics (6–8)

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| Expressions and Equations | Apply and extend previous understandings of arithmetic to algebraic expressions | - Write, read, and evaluate expressions in which letters stand for numbers.  
- Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient). View one or more parts of an expression as a single entity.  
- Evaluate expressions at specific values of their variables. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (order of operations).  
- Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). | CORE Math Academy for Middle Schools, Session 5  
Entering Algebra |
| Reason about and solve one-variable equations and inequalities | | - Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.  
- Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. Understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.  
- Solve real-world and mathematical problems by writing and solving equations of the form \( x + p = q \) and \( px = q \) for cases in which \( p, q, \) and \( x \) are all nonnegative rational numbers.  
- Solve linear equations in one variable. |
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| Expressions and Equations (cont.) | Use properties of operations to generate equivalent expressions             | • Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.  
• Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.                                                                 | CORE Math Academy for Middle Schools, Session 5  
Entering Algebra                                                                                                           |
| Solve real-life and mathematical problems using numerical and algebraic expressions and equations | Solve multistep real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.  
Apply properties of operations to calculate with numbers in any form, convert between forms as appropriate, and assess the reasonableness of answers using mental computation and estimation strategies.  
• Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. |                                                                                                                                                                                                 |                                                                                                                                 |
| Represent and analyze quantitative relationships between dependent and independent variables | Use variables to represent two quantities in a real-world problem that change in relationship to one another. Write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.  
• Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. |                                                                                                                                                                                                 |                                                                                                                                 |
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<td><strong>Geometry</strong></td>
<td><strong>Understand concepts of volume and relate volume to multiplication and to addition</strong></td>
<td>This session serves as an intervention for struggling adolescent learners focusing on understanding and working with two- and three-dimensional geometric shapes.</td>
<td>CORE Math Academy for Middle Schools, Session 4</td>
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|                   | **Classify two-dimensional figures into categories based on their properties** | - Know relative sizes of measurement units within one system of units.  
- Measure areas by counting unit squares.  
- Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.  
- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.  
- Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.  
- Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.  
- Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category.  
- Classify two-dimensional figures in a hierarchy based on properties   | Geometry and Measurement                          |
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| **Geometry (cont.)** | Solve real-world and mathematical problems involving area, surface area, and volume | • Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes.  
• Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths of the prism.  
• Represent three-dimensional figures using nets and use the nets to find the surface area of these figures. | CORE Math Academy for Middle Schools, Session 4  
Geometry and Measurement |
|                      | Solve real-life math problems involving angle, surface area, and volume | • Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. | |
| Understand congruence and similarity using physical models, transparencies, or geometry software | • Verify experimentally the properties of rotations, reflections, and translations. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations.  
• Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. | |
| Understand and apply the Pythagorean Theorem | • Explain a proof of the Pythagorean Theorem and its converse.  
• Apply the Pythagorean Theorem to determine unknown side length in right triangles in real-world and mathematical problems in two and three dimensions. | |